Frequency Space Environment Map Rendering

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ttp://graphics.stanford.edu/papers/freqen

Demo

Motivation: Interactive rendering with complex natural illumination and realistic, measured BRDFs

Reflection Equation

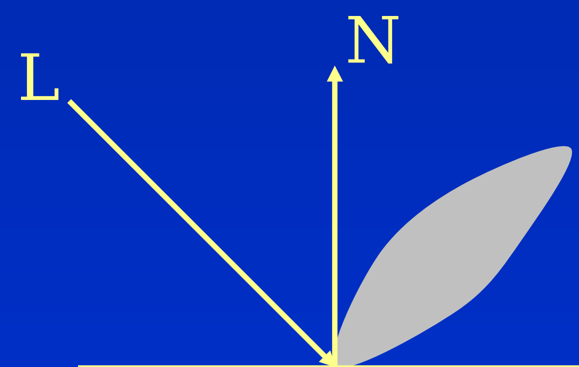
$$L\left(R(\vec{N})\,\vec{l} \right)$$

2D Environment Map

Reflection Equation

$$L\left(R(\vec{N})\,\vec{l}\right)\, r\left(\vec{l},\vec{V}\right)$$

2D Environment Map BRDF



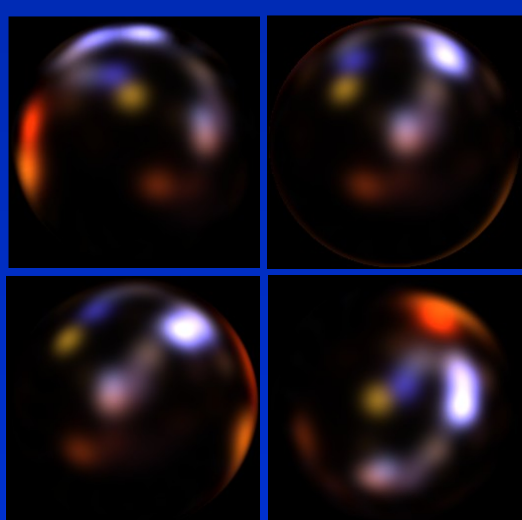
Reflection Equation

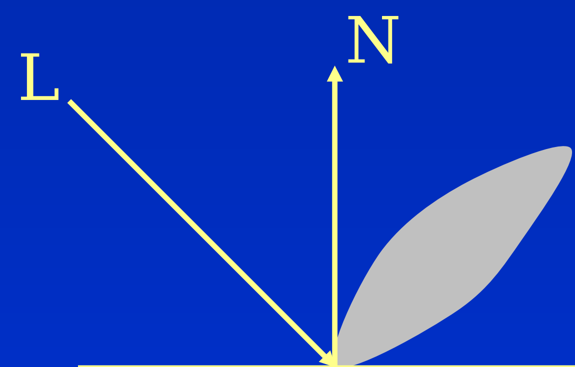
$$B(\vec{N},\vec{V}) = \int_{\Omega} L\left(R(\vec{N})\vec{\lambda}\right) \rho\left(\vec{\lambda},\vec{V}\right) d\lambda$$

4D Orientation Light Field

2D Environment Map

BRDF





Previous Work: Blinn & Newell 76, Miller & Hof
Greene 86, Kautz & McCool 99, Cabral et al. 99

Goals

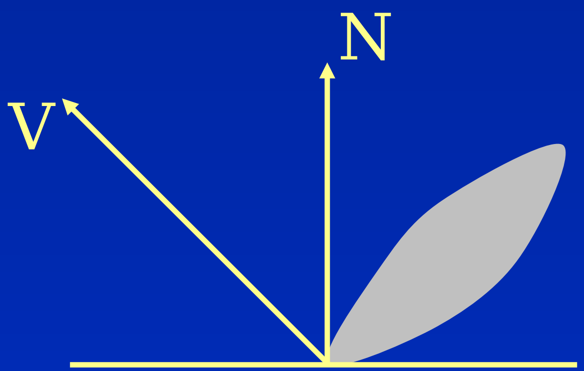
- Efficiently precompute and represent OLF
- Real-time rendering with OLF

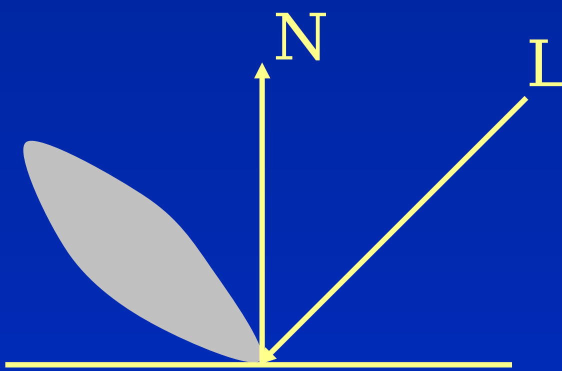
Questions

- Parameterization and structure of OLF
- Structure leads to representation
- Computation and rendering of OLF

OLF Parameterization

$B(\vec{N},\vec{V})$

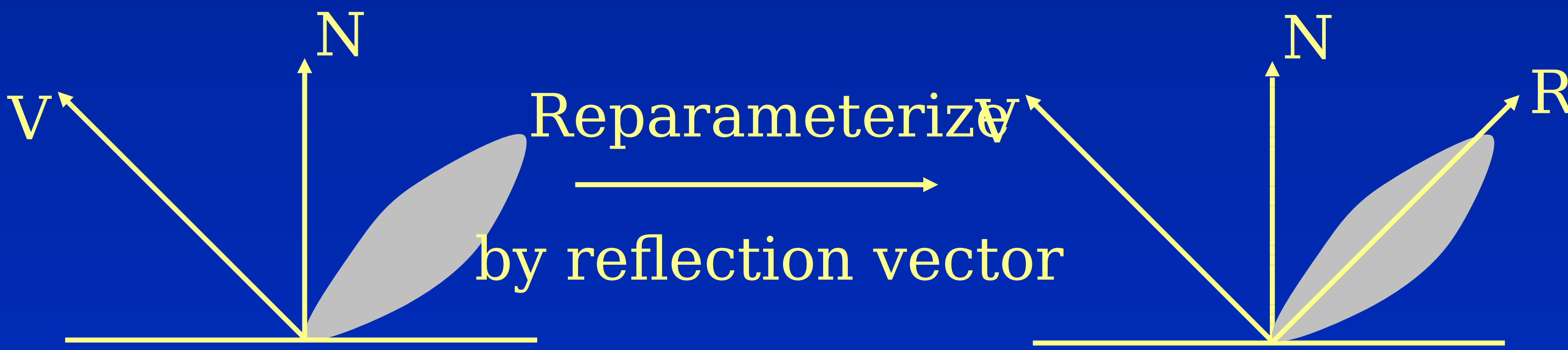




OLF Parameterization

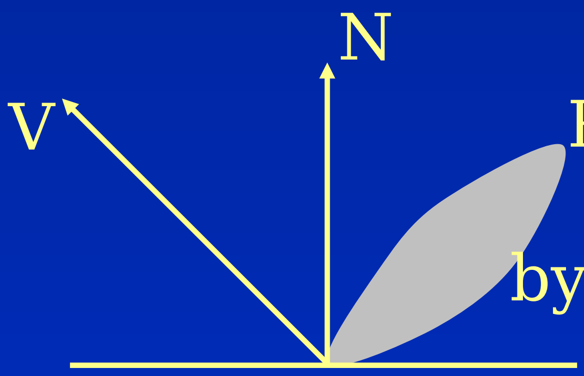
$$B(\vec{N},\vec{V})$$

$$B(\vec{R},\vec{V})$$



OLF Parameterization

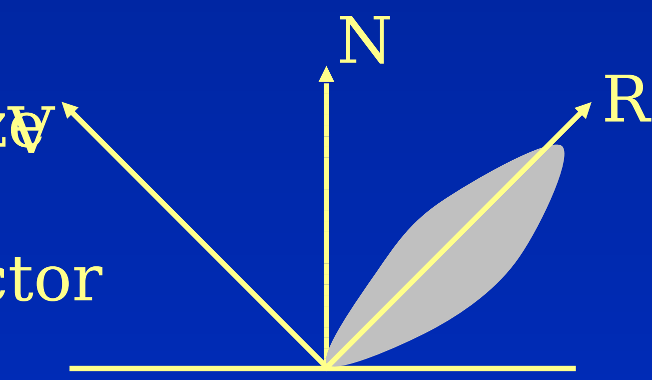
$B(\vec{N},\vec{V})$



Reparameterize

by reflection vector

$B(\vec{R},\vec{V})$



- Captures structure of BRDF (and hence OLF) better
- Reflective BRDFs become low-dimensional

OLF Structure

$$B(\vec{R},\vec{V})$$

$$B_V(\vec{R})$$

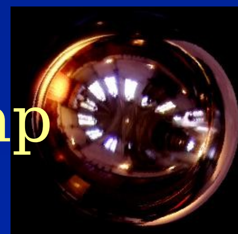
2D view array of reflection maps

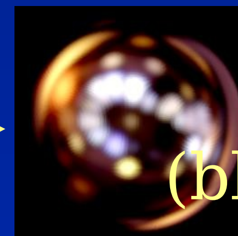
$$B_R(\vec{V})$$

2D image array of view maps

OLF Structure: Phong

Environment Map



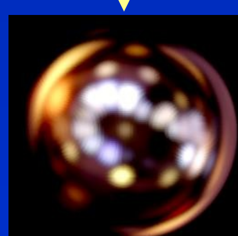


Phong Reflection Map
(blurred environment ma

$$B(\vec{R},\vec{V}) = B(\vec{R})$$

$$B_V(\vec{R})$$

2D view array of
reflection maps



ame reflection map for all views

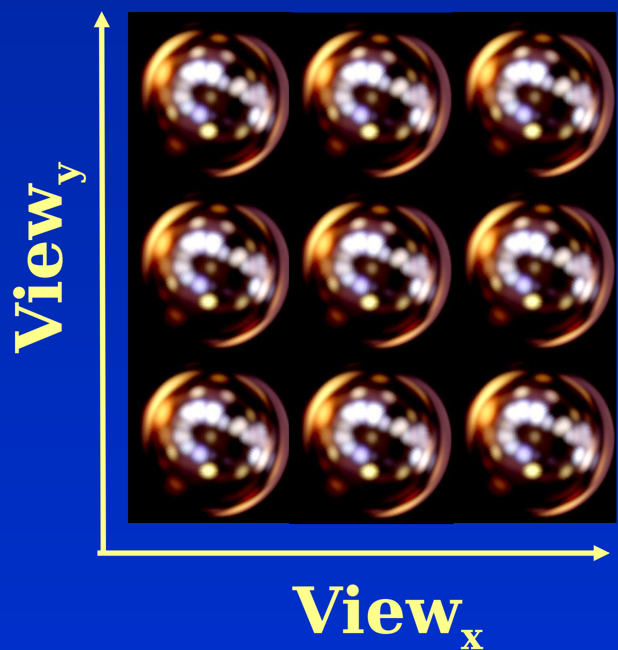
$$B_R(\vec{V})$$

2D image array
of view maps

OLF Structure: Phong

$$B(\vec{R},\vec{V}) = B(\vec{R})$$

$$B_V(\vec{R}) \qquad\qquad B_R(\vec{V})$$

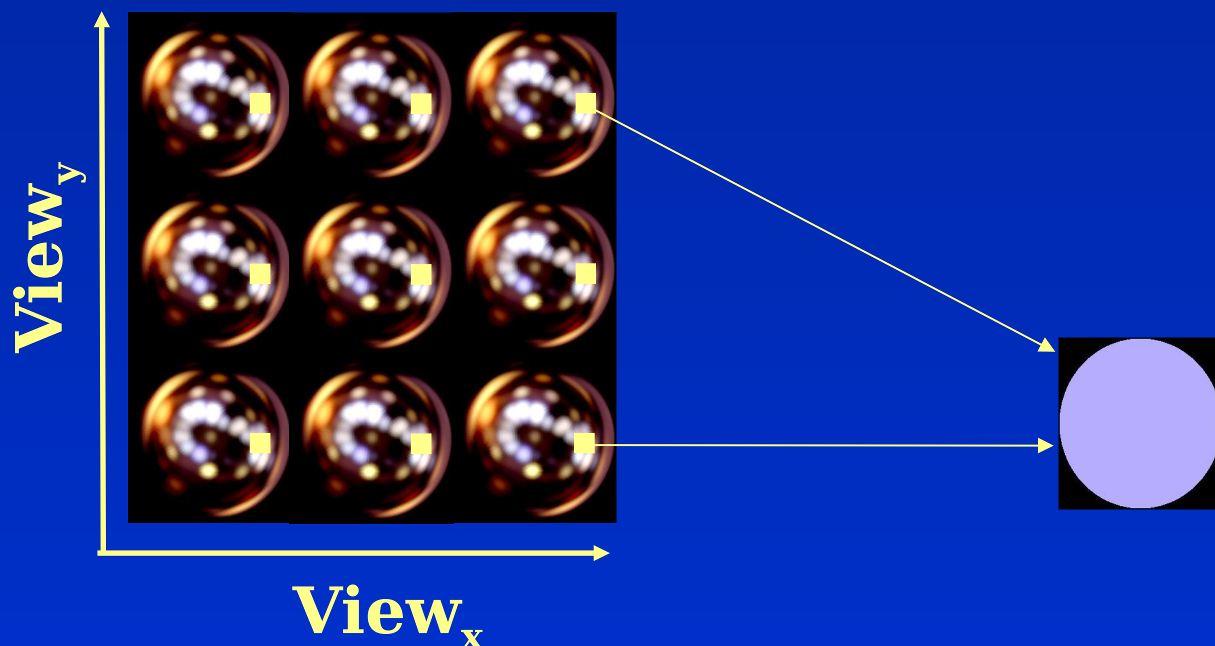


ame reflection map for all views

OLF Structure: Phong

$$B(\vec{R},\vec{V}) = B(\vec{R})$$

$$B_V(\vec{R}) \qquad\qquad B_R(\vec{V})$$



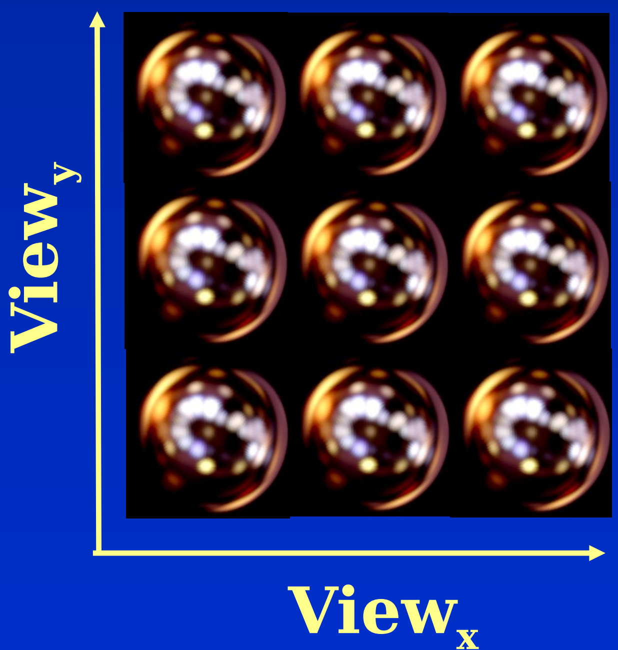
ame reflection map for all views

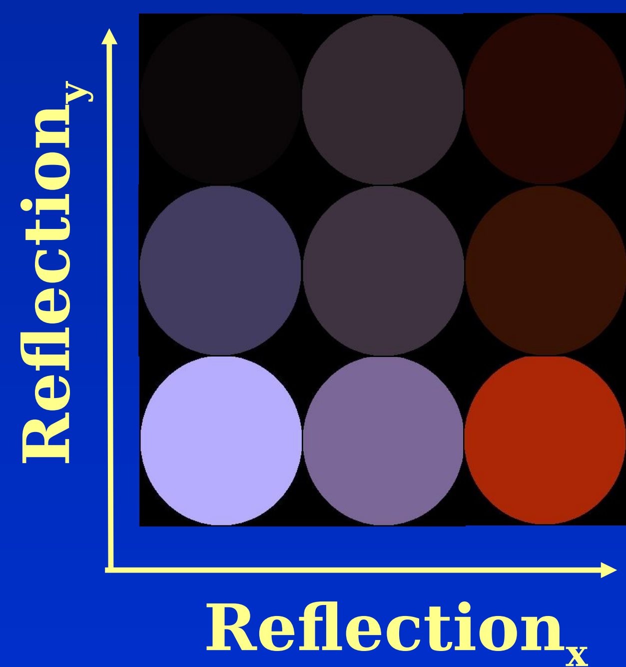
View maps constant for each R

OLF Structure: Phong

$$B(\vec{R},\vec{V}) = B(\vec{R})$$

$$B_V(\vec{R}) \qquad\qquad B_R(\vec{V})$$



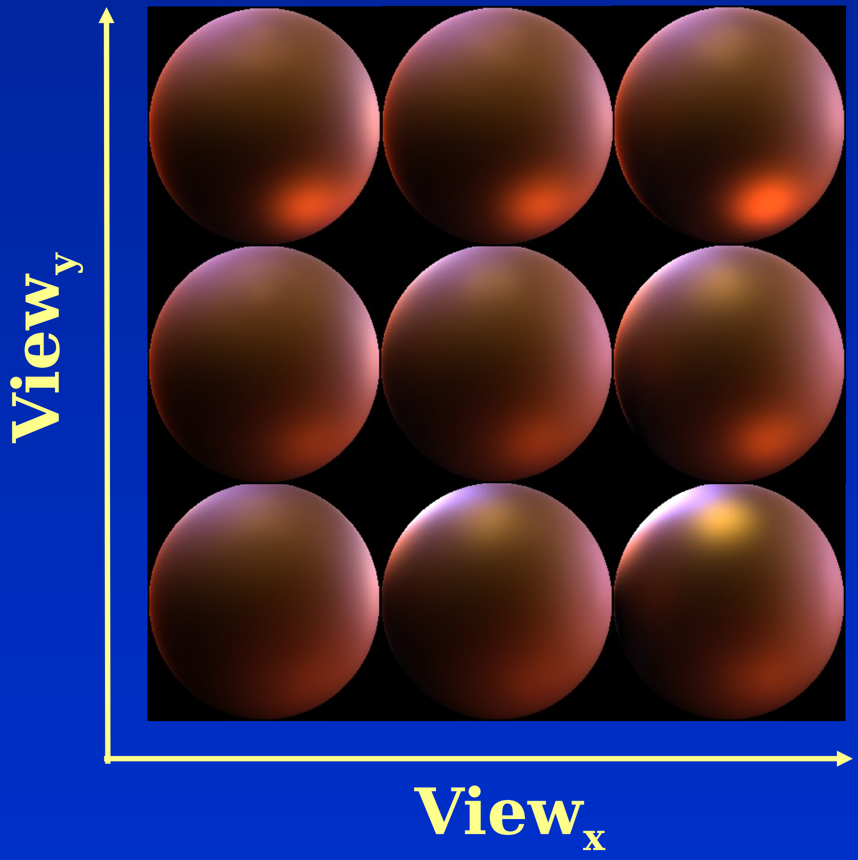


ame reflection map for all views

View maps constant for each R

OLF Structure: Lafortune

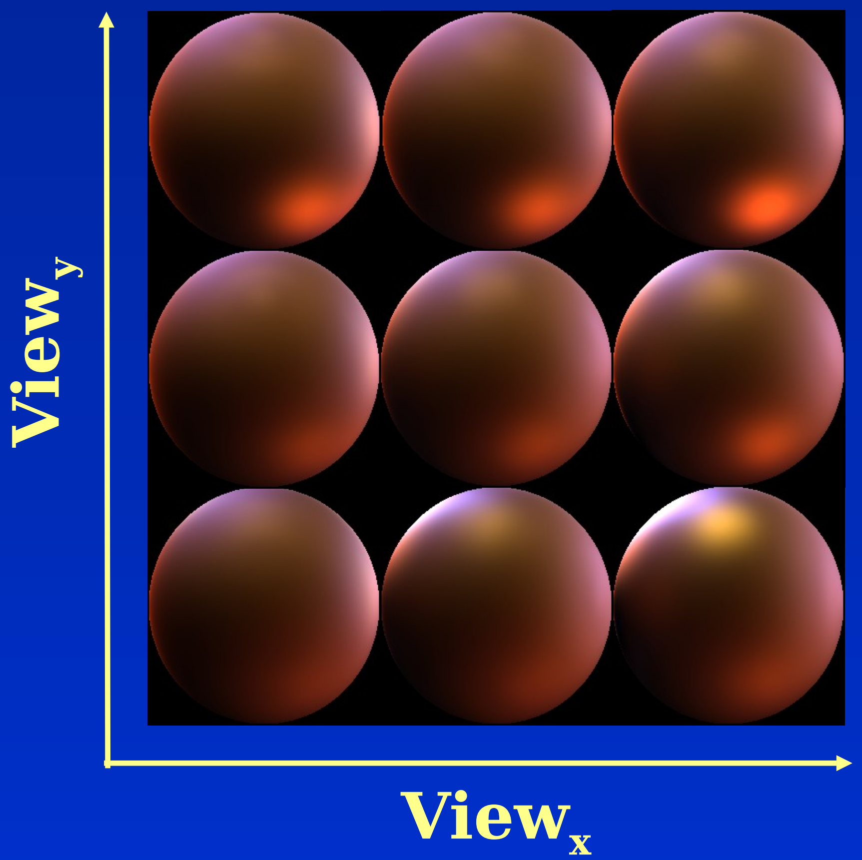
$$B_V(\vec{R})$$



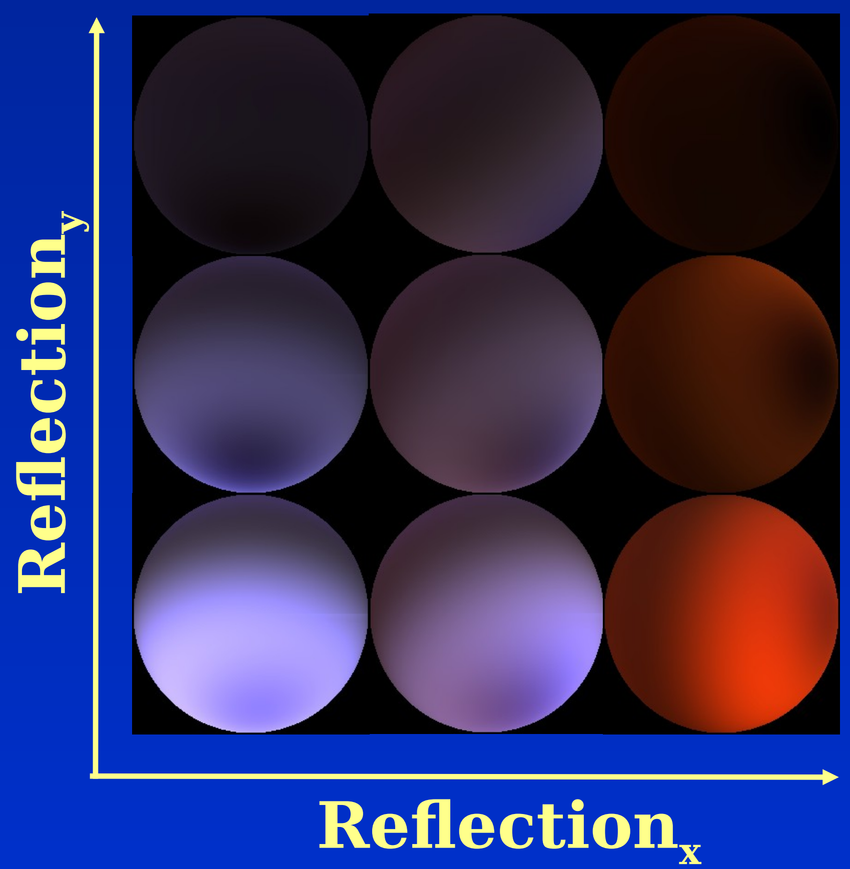
- Single 2D reflection map no longer sufficient
- But variation with viewing direction is slow

OLF Structure: Lafortune

$$B_V(\vec{R})$$



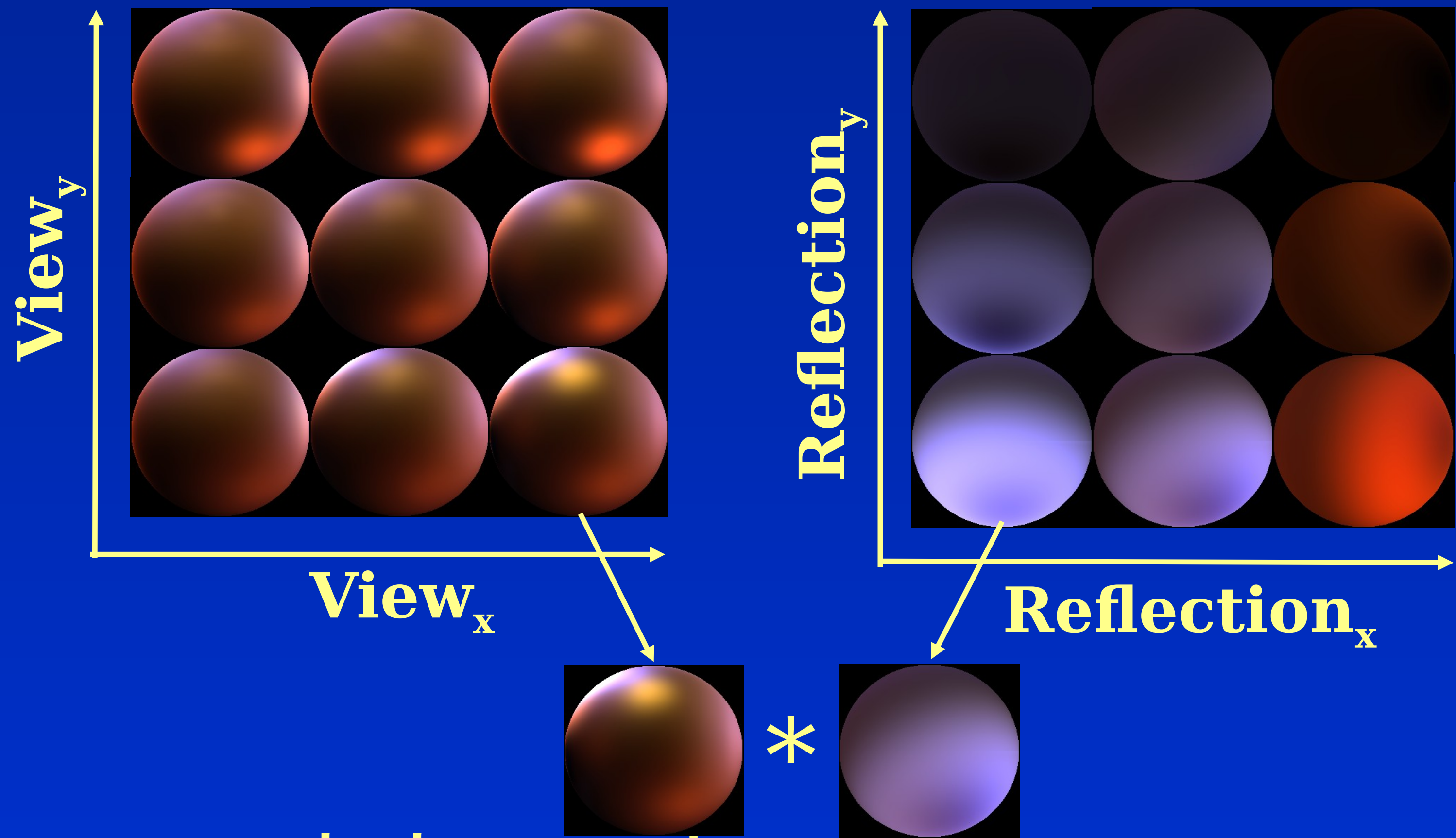
$$B_R(\vec{V})$$



View maps vary slowly

A Simple Factorization

$$B_{V}(\vec{R}) \qquad\qquad B_{R}(\vec{V})$$



$$B(\vec{R},\vec{V}) \approx f(\vec{R}) * g(\vec{V})$$

Questions

- Parameterization and structure of OLF
- Structure leads to representation
  - Frequency space analysis
- Computation and rendering of OLF

Convolution

$$B(\vec{N},\vec{V}) = \int_{\Omega} L\left(R(\vec{N})\,\vec{l} \right) \rho\left(\vec{l},\vec{V} \right) dl$$

$$B = L \otimes \rho$$

Spherical harmonic analysis

$$B_{ij} = L_i \rho_{ij}$$

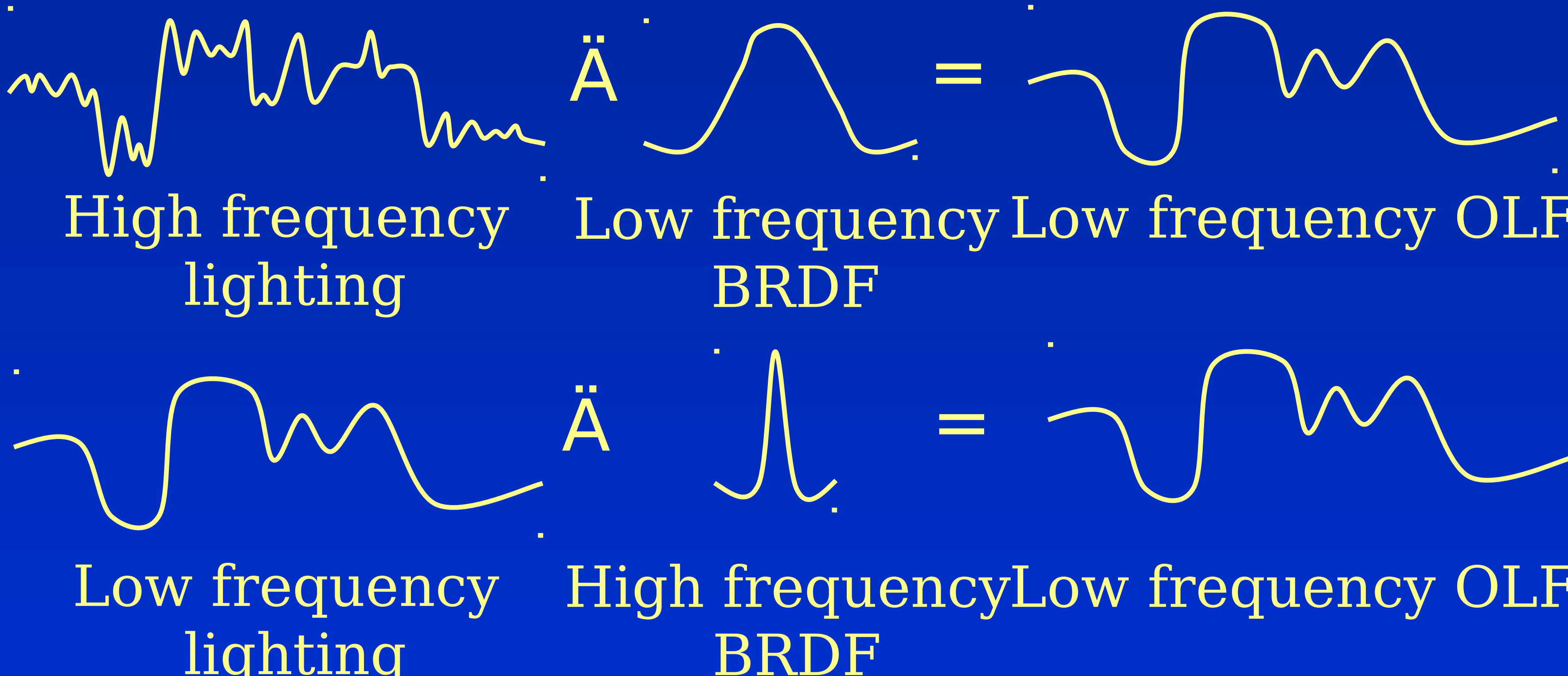
Spatial: integral

Frequency: product

Ramamoorthi and Hanrahan

Implications

- Information content of OLF determined by information in lighting and BRDF

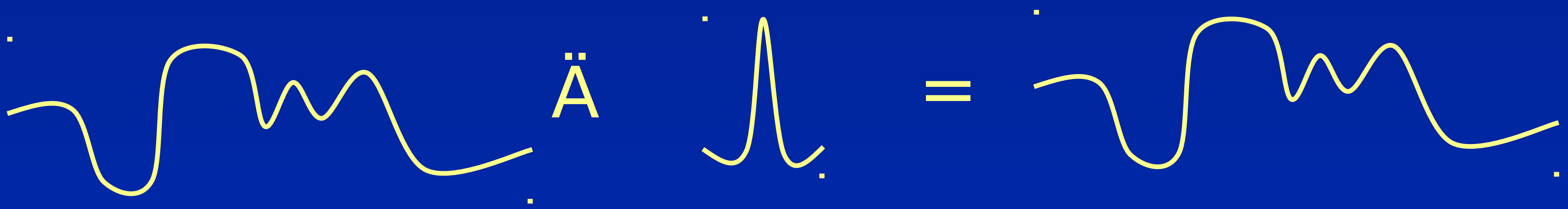


Implications

Sampling rates/resolutions

- Minimum of highest light, BRDF frequencies
- Angular resolution proportional to max frequency

Example: Low frequency L



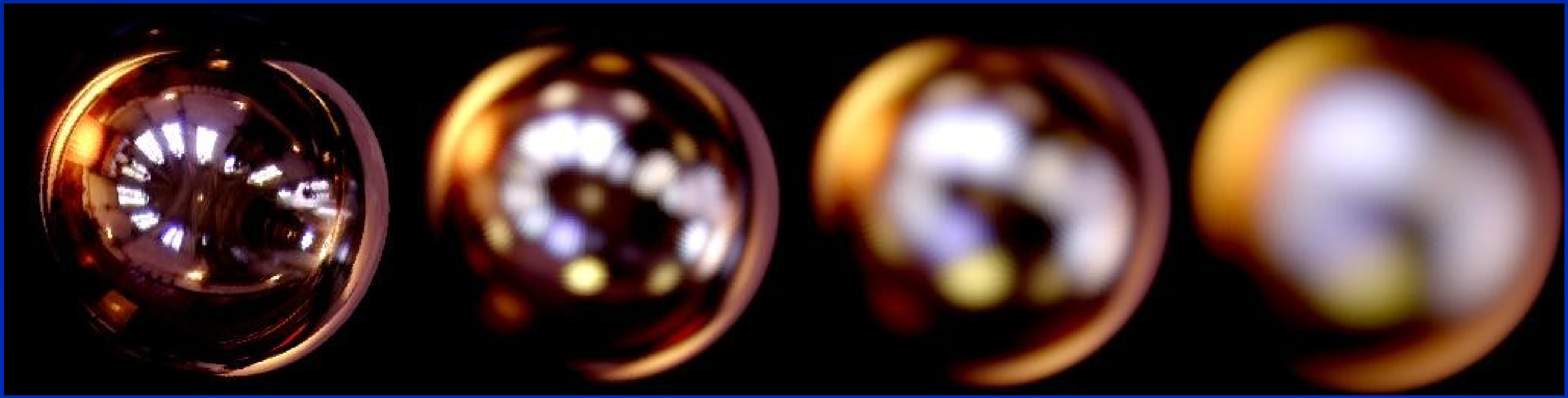
Low frequency lighting　High frequency BRDF　Low frequency OLF

Example: Low frequency lighting [Sloan et al. 02]

- OLF is low frequency
- Represent with low-order spherical harmonics only
- Compute OLF using coefficient multiply
  [Cabral et al. 87, Kautz et al. 02]

Natural Lighting

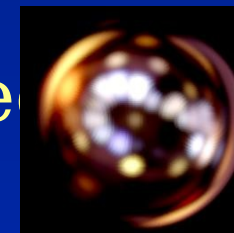
Natural (high frequency) lighting



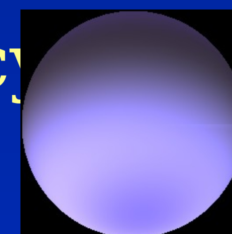
4000 terms 400 terms 100 terms 36 terms

Hybrid Representation

- Reflection maps $B_V(\vec{R})$ are high frequency



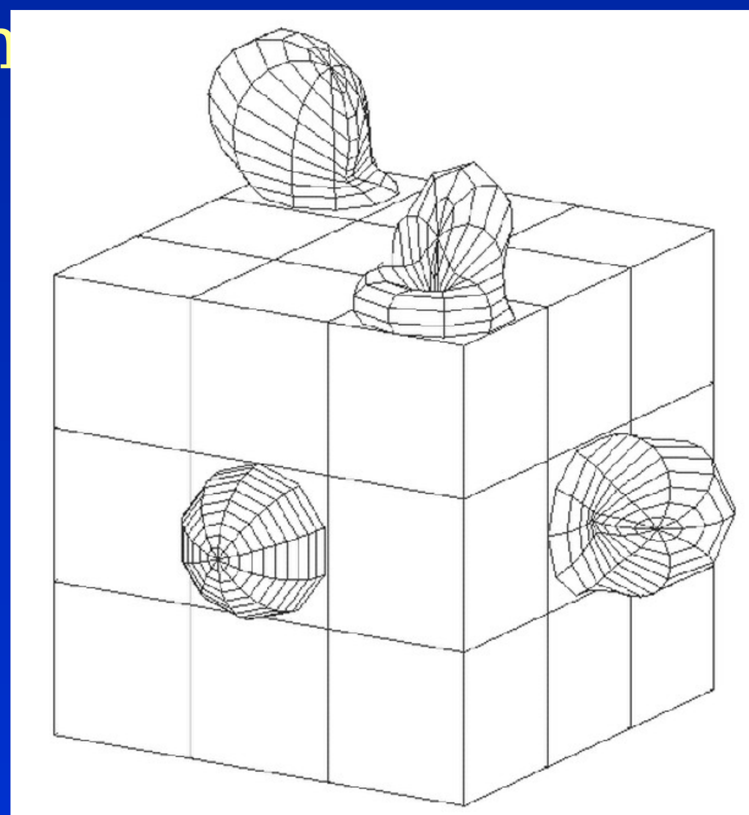
- View maps $B_R(\vec{V})$ are low frequency



- Use hybrid angular frequency-space representation
  - View maps: Use low-order spherical harmonic expansion
  - Represent coefficient reflection maps explicitly

Spherical Harmonic Reflection Map

- View-dependent reflection (cube)map
- Encode view maps $B_R(\vec{V})$ with low-order spherical h

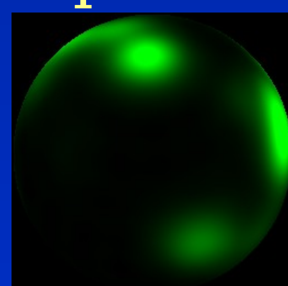


Spherical Harmonic Reflection Map

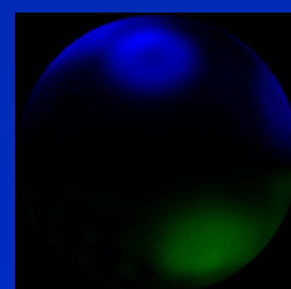
$$B(\vec{R},\vec{V}) = \sum_{i=0}^{N} B_i(\vec{R})\, Y_i(\vec{V})$$

Spherical Harmonics

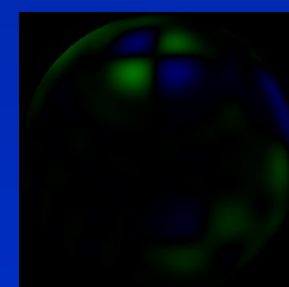
Precomputed coefficient reflection maps



B_0



B_1



B_2

Questions

- Parameterization and structure of OLF
- Structure leads to representation
- Computation and rendering of OLF

Prefiltering

$$L, r \longrightarrow \boxed{L_i, r_{ij} \longrightarrow B_{ij} = L_i r_{ij}} \longrightarrow B_i(R)$$

Input Lighting and BRDF	Spherical Harmonic coeffs.	Convolution	SHRM

- Directly compute SHRM from Lighting, BRDF
- Convolution easier to compute in frequency domain

Prefiltering

$$L, r \longrightarrow \boxed{L_i, r_{ij} \longrightarrow B_{ij} = L_i r_{ij}} \longrightarrow B_i(R)$$

Spherical Harmonic coeffs.

Convolution

- 3 to 4 orders of magnitude faster (< 1 s compared to minutes or hours)
- Detailed analysis, algorithms, experiments in paper

SHRM Rendering

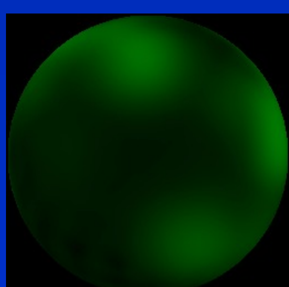
We create dynamic reflection map per frame

- Weighted sum of prefiltered coefficient reflection maps

$$B_{\vec{V}}(\vec{R}) = \sum_{i=0}^{N} Y_i(\vec{V})\, B_i(\vec{R})$$

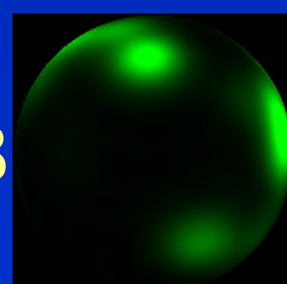
Spherical Harmonics (fixed weighting factor)

Prefiltered coefficient reflection maps

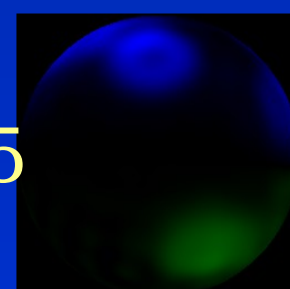


=

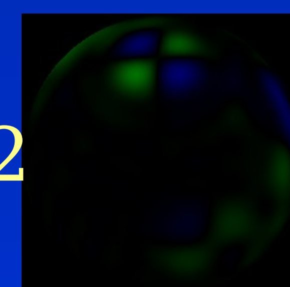
.3



+.5



+.2



Y_0 B_0 Y_1 B_1 Y_2 B_2

Number of SHRM terms

Microfacet model, roughness = .2

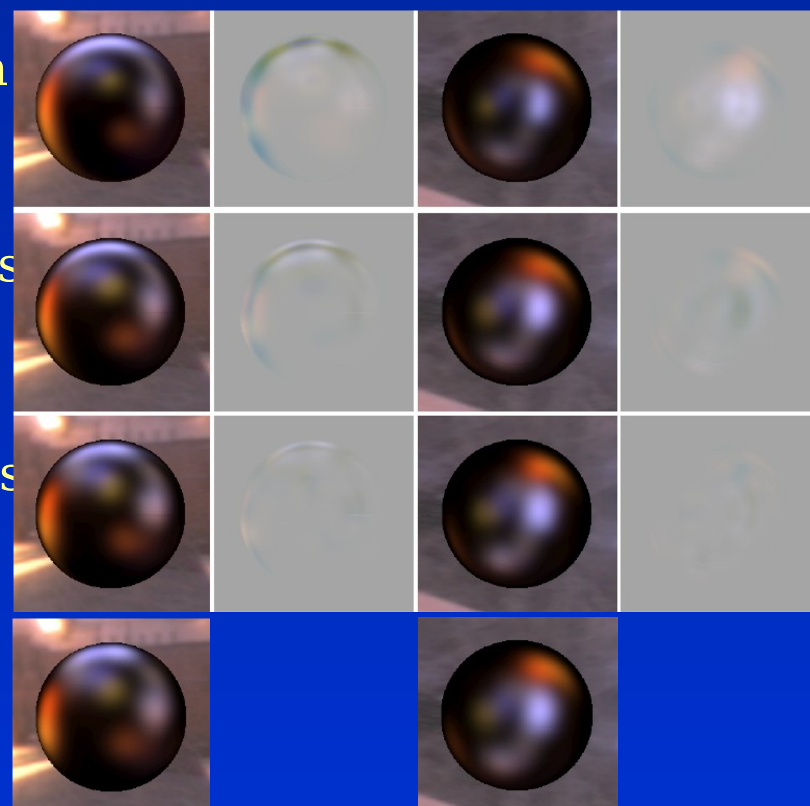
View 1 | Difference Image | View 2 | Difference Image

1 term 88%

4 terms 95%

9 terms 98%

Exact 100%



Number of terms: CUReT

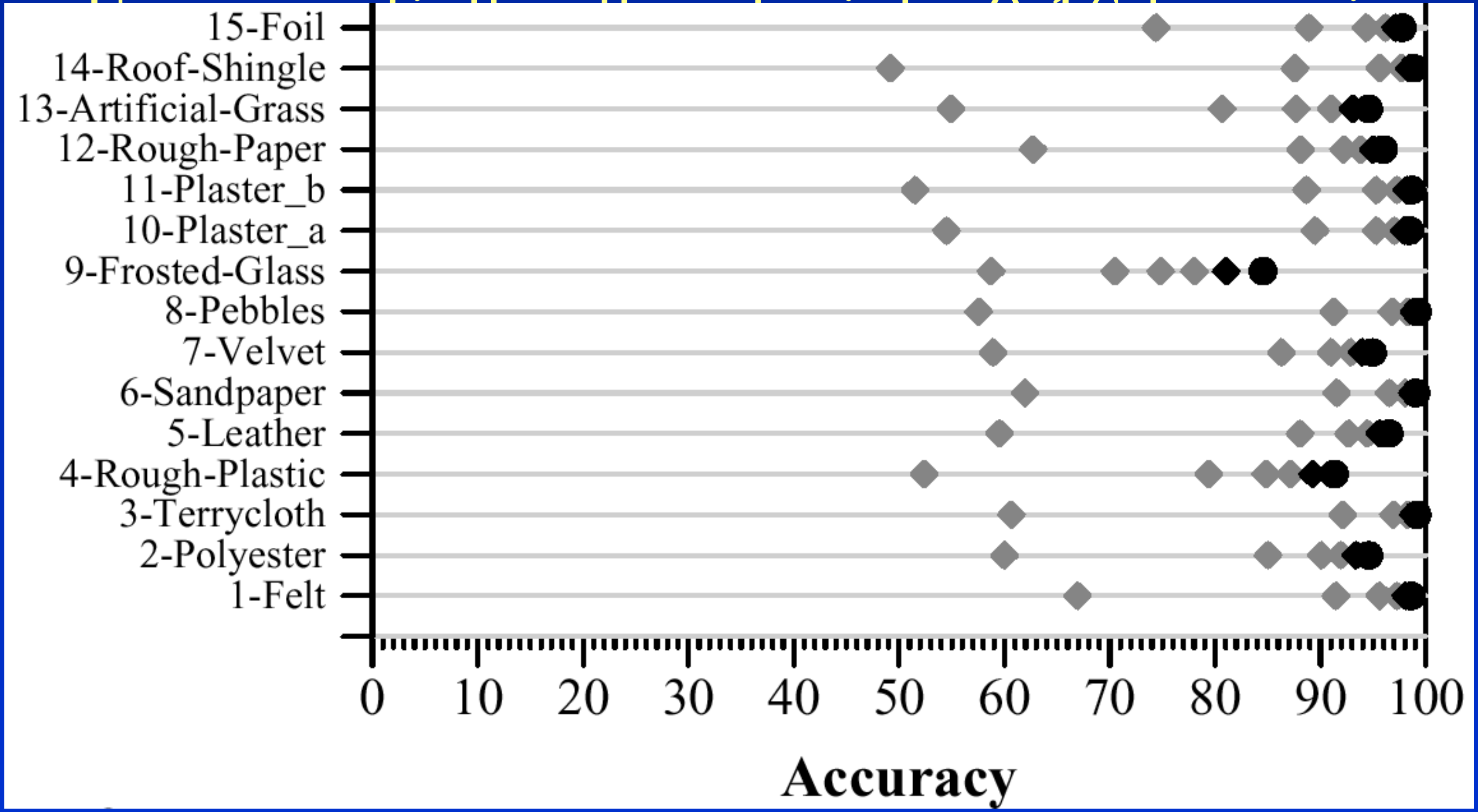
CUReT database [Dana et

- 61 BRDFs of real material
  205 measurements for ea

- Interpolated using order 8
  Zernike polynomials
  [Koenderink, van Doorn 9

Number of terms: CURET

Analysis for all 61 samples [full bar chart in paper]



Implementation

- Stanford Real-Time Programmable Shading System
- SHRMs used in any shader just like reflection map
- New reflection map computed for each frame
- Real-time (>15Hz) performance on 1.4 GHz Pentium IV with nVidia Geforce 2
- http://graphics.stanford.edu/papers/freqenv/

Demo

Summary of Contributions

- Theoretical, empirical analysis of sampling rates and resolutions
  - Frequency space analysis directly on lighting, BRDF
  - Low order expansion suffices for essentially all BRDFs
- Spherical Harmonic Reflection Maps
  - Hybrid angular-frequency space
  - Compact, efficient, accurate
  - Easy to analyze errors, determine number of terms
- Fast computation using convolution

Implications and Future Work

- Frequency space methods for rendering
  - Global illumination
  - Fast computation of surface light fields
- Compression for optimal factored representations
  - PCA on SHRMs
- Theoretical analysis of sampling rates, resolutions
  - General framework for sampling in image-based rendering

Acknowledgements

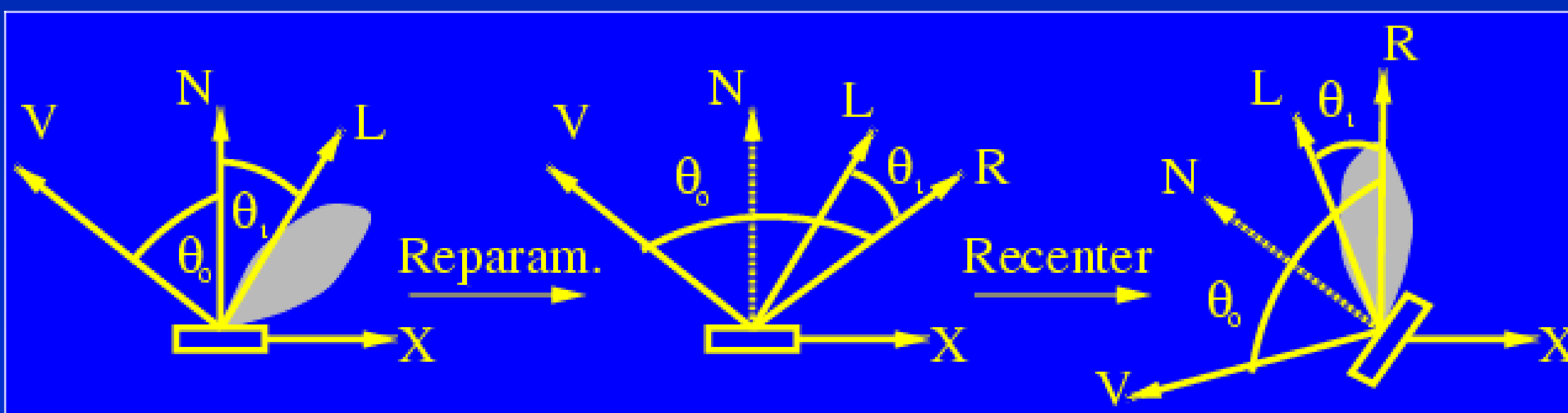
- Stanford Real-Time Programmable Shading System
  - Eric Chan, Bill Mark, Kekoa Proudfoot
- Readers of early drafts
  - Li-Yi Wei, Olaf Hall-Holt, anonymous reviewers
- Models
  - Armadillo: Venkat Krishnamurthy
  - Light probes: Paul Debevec
- Funding
  - Hodgson-Reed Stanford Graduate Fellowship
  - NSF ITR #0085864 “Interacting with the Visual World”

The End

BRDF Parameterization

BRDF

- Direct $$\rho(\vec{\omega}_i,\vec{\omega}_o) \approx f(\vec{\omega}_i)\,g(\vec{\omega}_o)$$
- Half Angle $$\rho(\vec{\omega}_h,\vec{\omega}_i,\vec{\omega}_o) \approx f(\vec{\omega}_i)\,g(\vec{\omega}_h)\,f(\vec{\omega}_o)$$
  [Rusinkiewicz 98, McCool et al. 01]
- Reflection Vector $$\rho(\vec{\omega}_i^R,\vec{\omega}_o^R) \approx f(\vec{\omega}_i^R)\,g(\vec{\omega}_o^R)$$



Parameterization

- Lighting: 2D function on a sphere $L(\vec{\omega}_i)$
- BRDF
  - Direct
  - Half Angle
  - ***Reflection Vector***

$$\rho(\vec{\omega}_i^R, \vec{\omega}_o^R)$$

- OLF
  - Direct
  - No Half Angle
  - ***Reflection Vector***

$$B(\vec{R}, \vec{V})$$

OLF Parameterization

- Direct $$B(\vec{N},\vec{\omega}_o) : f(\vec{N})g(\vec{\omega}_o)$$
- Reflection Vector (reflection, normal, view)
  - Captures structure of BRDF and OLF
  - Reflective BRDFs, OLFs become low-dimensional

$$B(\vec{R},\vec{N},\vec{V}) : f(\vec{R})g(\vec{N})h(\vec{V})$$

$$B(\vec{R},\vec{N}) : f(\vec{R})g(\vec{N})$$

$$B(\vec{R},\vec{V}) : f(\vec{R})h(\vec{V})$$

Latta and Kolb 02
Wood et al. 00

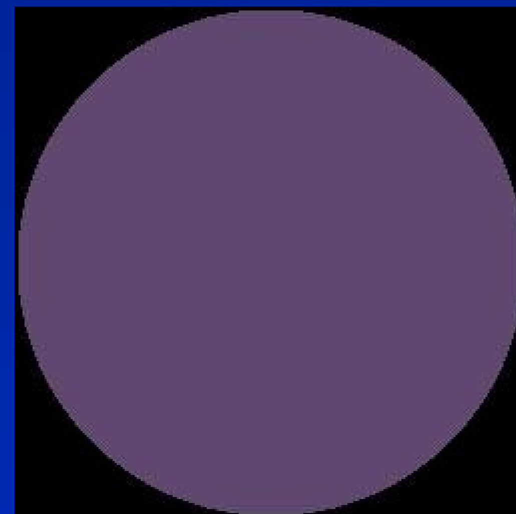
Advantages

- Good param. for *both* BRDF, O
- Fast computation with convolu
- Single reflection map for each

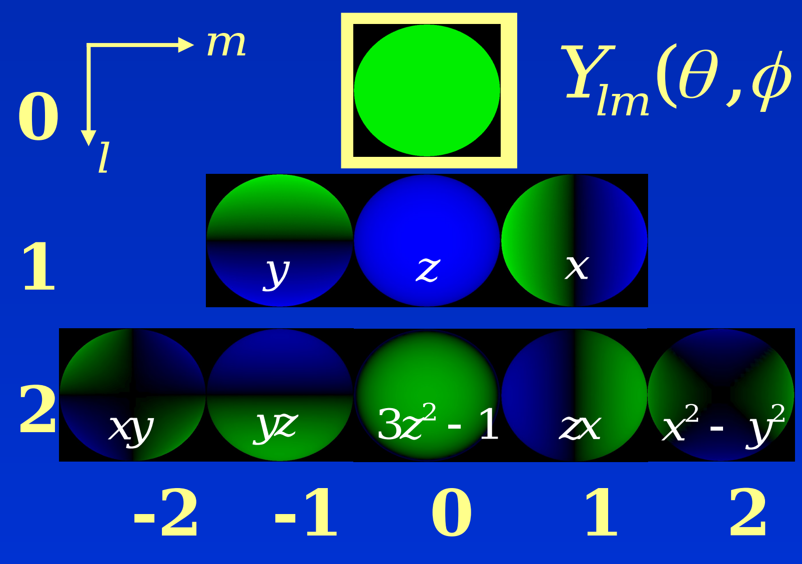
SHRM approximation

Exact Image

$B_R(\vec{V})$



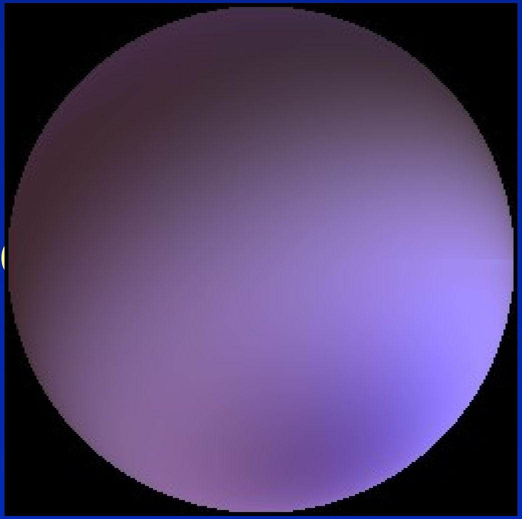
Order 0
1 term



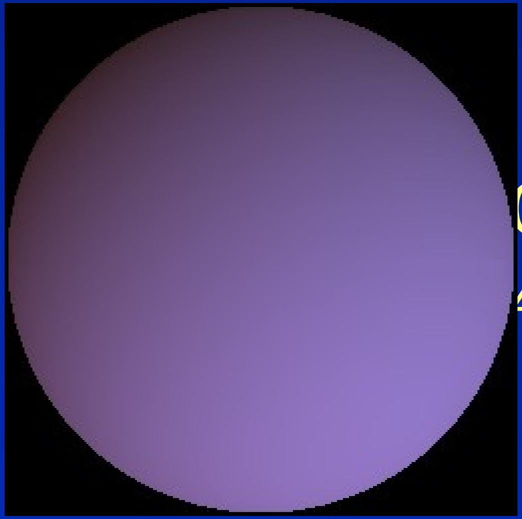
$Y_{lm}(\theta,\phi)$

SHRM approximation

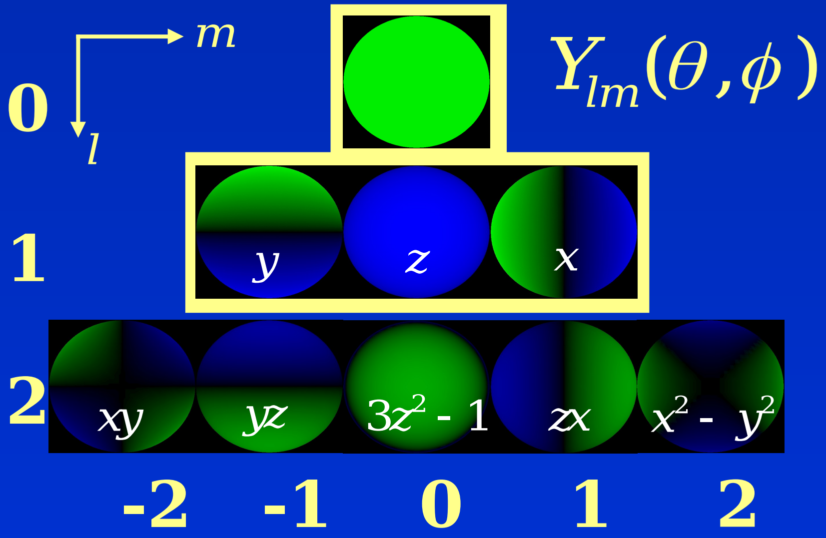
Exact Image



$B_R(\vec{V})$



Order 1
4 terms



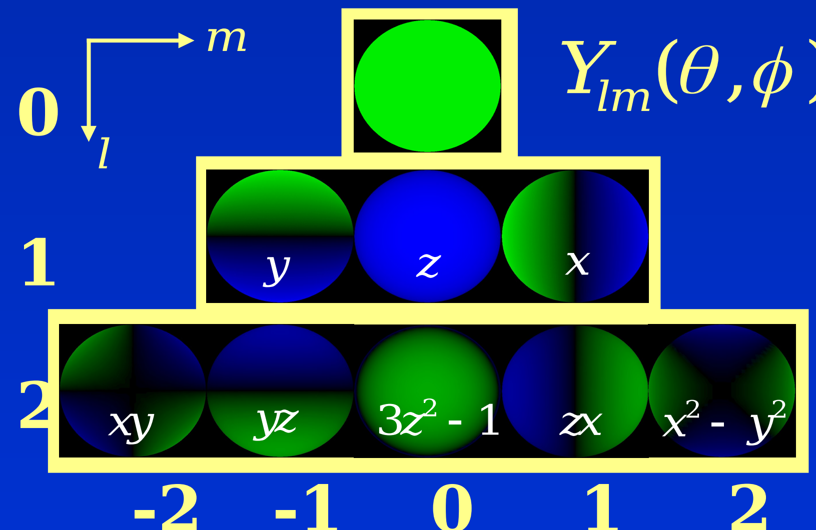
$Y_{lm}(\theta,\phi)$

SHRM approximation

Exact Image

Order 2
9 terms

$B_R(\vec{V})$



$Y_{lm}(\theta, \phi)$

Example: Phong BRDF

$$C_f = O(S^2\sqrt{s})$$ Frequency

Cost

$$C_a = O\left(S^4/s\right)$$ Angular

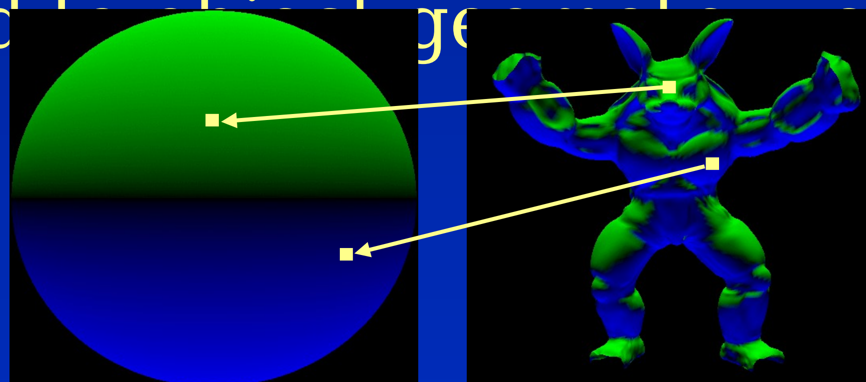
S = resolution, s = Phong exponent

Frequency space faster unless s > 500

Usually 3 to 4 orders of magnitude faster
(< 1 s compared to minutes or hours)

Orientation Light Field

- 4D function of surface normal, viewing direction
- Mapped to object geometry using surface normal



Reflection Equation

$$B(\vec{N},\vec{\omega}_o) = \int_{\Omega} L\left(R(\vec{N})\,\vec{\omega}_i\right)\,\rho\left(\vec{\omega}_i,\vec{\omega}_o\right)\,d\omega_i$$

Reflected Radiance (4D Orientation Light Field)

Distant Lighting (2D Environment Map)

Isotropic BRDF

$$B = L \otimes \rho$$

Basri and Jacobs 01

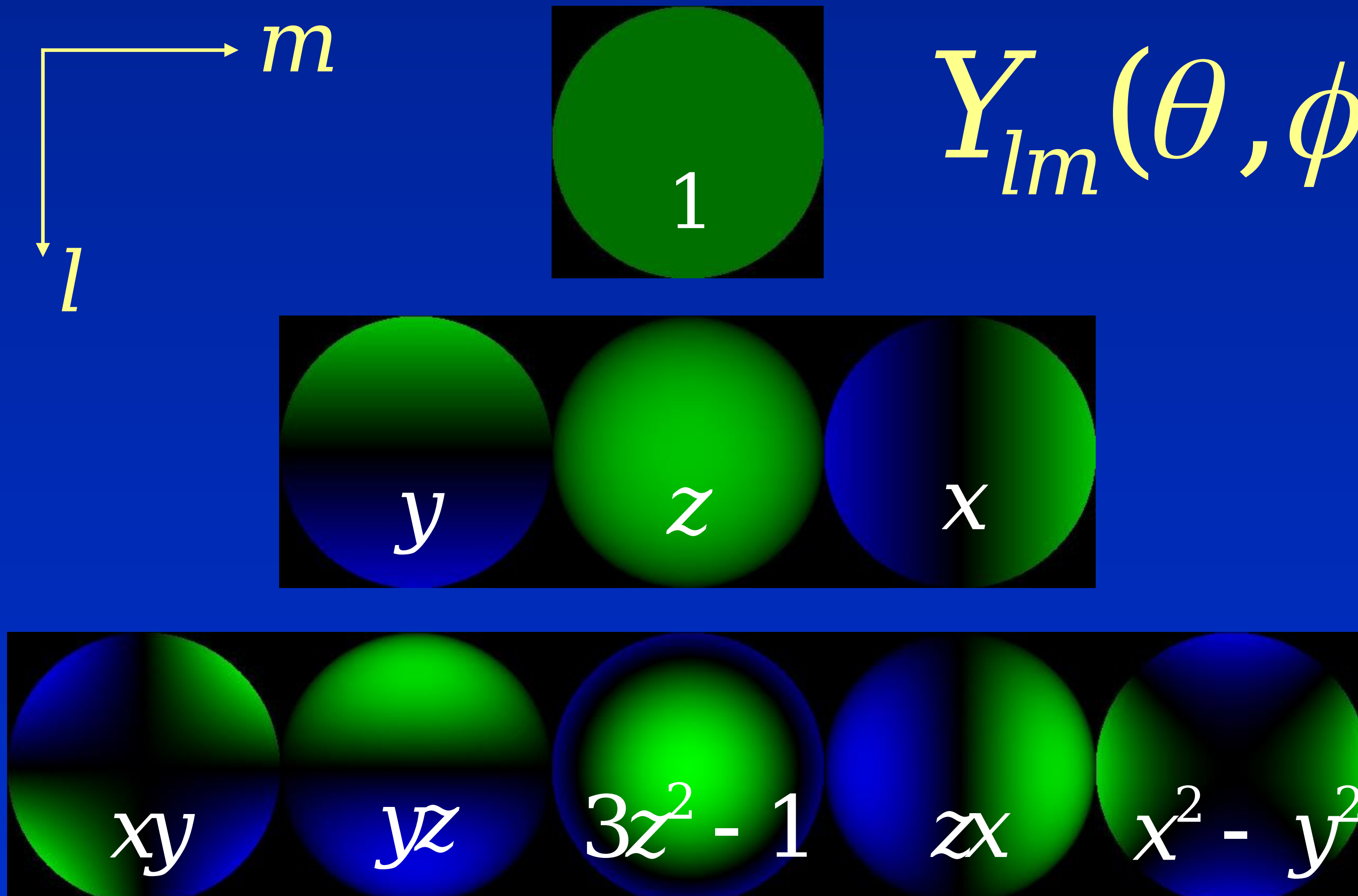
Ramamoorthi and Hanraha

Spherical Harmonics

$$Y_{lm}(\theta,\phi)$$

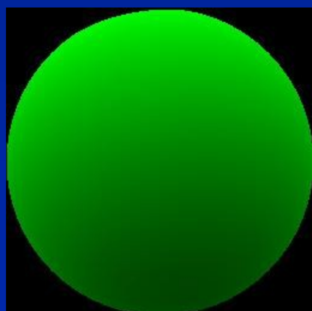
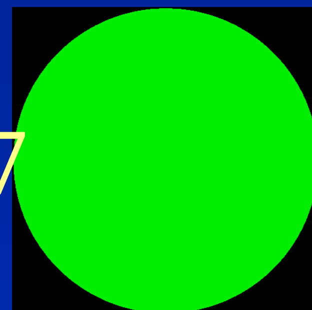
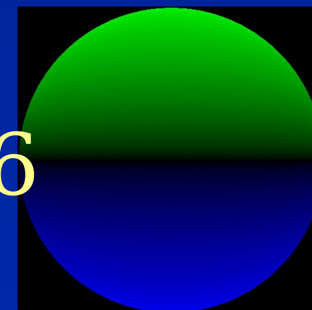
m

l



l \ m	-2	-1	0	1	2
0			1		
1		y	z	x	
2	xy	yz	$3z^2 - 1$	zx	$x^2 - y^2$
.					
.					
.					

Spherical Harmonic expansion

 = .67  + .36  + ...

Expand Lighting, BRDF, OLF in spherical harmonics

$$L(\theta,\varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} L_{lm} Y_{lm}(\theta,\varphi)$$

Convolution

- Lighting $L(\vec{\omega}_i)$ coefficients L_{lm}
- BRDF $\rho(\vec{\omega}_i^R, \vec{\omega}_o^R)$ $\hat{\rho}_{lq,pq}$
- OLF $B(\vec{R}, \vec{V})$ $B_{lm,pq}$

$$B = L \otimes \rho$$

$$B_{lm,pq} = L_{lm} \hat{\rho}_{lq,pq}$$

Ramamoorthi and Hanrahan

This Session

Latta and Kolb: Homomorphic single-term factorization

- Advantages of SHRMS: more accurate, easier to analyze errors/set resolutions, fast computation using convolution
- Disadvantage: Multi-term, fixed parameterization.
- Future work: compute best single-term approximation, or other factorizations directly from SHRM using PCA

This Session

Sloan et al., Kautz et al: Low frequency lighting

Advantages of SHRMS

- General lighting environments, BRDFs
- Error analysis determines number of terms
- Rapid computation

Disadvantage: As yet, no shadows, interreflection

Results

- SHRM accuracy: comparisons with previous methods (Cabral et al. 99, Kautz and McCool 00) in paper
- Speed of prefiltering: speedups of 3 to 4 orders of magnitude; times in fractions of a second
- Real-time rendering even with multiple SHRMs

Video

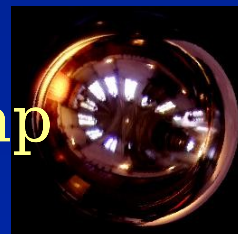

Lambertian
Krylon Blue
Anisotropic
rotateview
constrain
scene: armadillos2

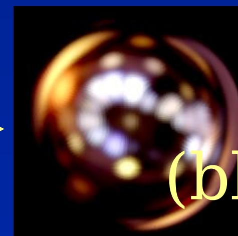
Video


Stanford Programmable Shading System
Lambertian
Krylon Blue
Anisotropic

OLF Structure: Phong

Environment Map



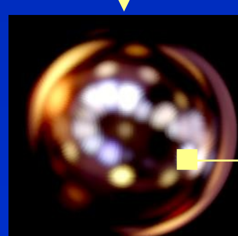


Phong Reflection Map
(blurred environment ma

$$B(\vec{R},\vec{V}) = B(\vec{R})$$

$$B_V(\vec{R})$$

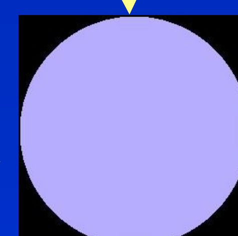
2D view array of
reflection maps



ame reflection map for all views

$$B_R(\vec{V})$$

2D image array
of view maps

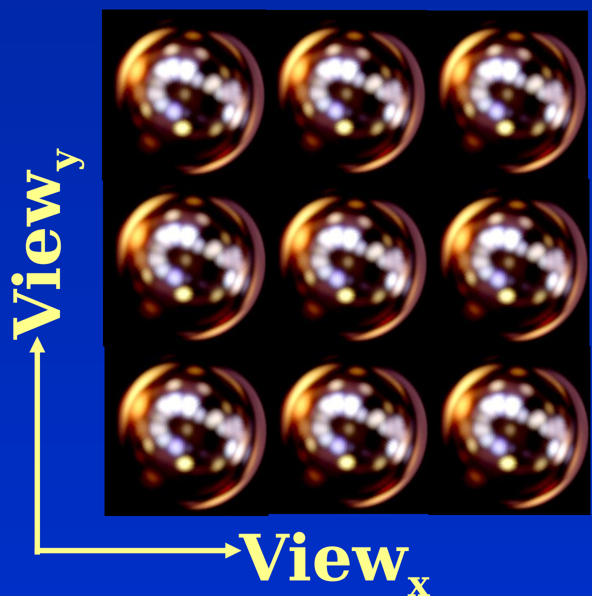


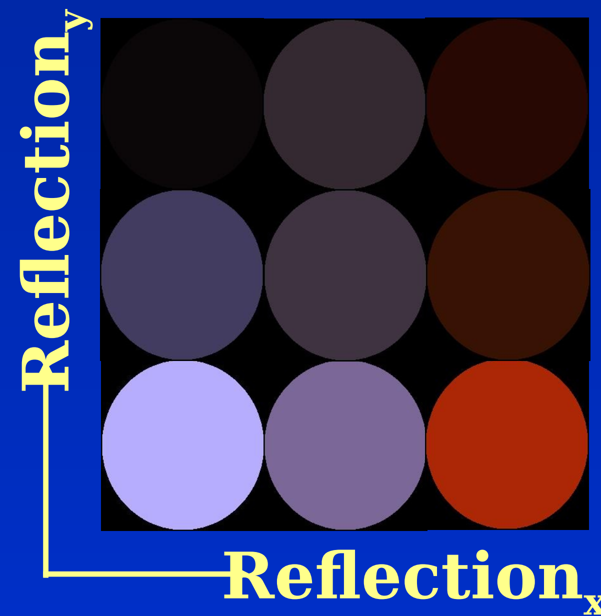
Different constant for each R

OLF Structure: Phong

$$B(\vec{R},\vec{V}) = B(\vec{R})$$

$$B_V(\vec{R}) \qquad\qquad B_R(\vec{V})$$





ame reflection map for all views

Hemispheres constant for each

Previous Work

Environment Maps

- Blinn & Newell 76, Miller & Hoffman 84, Greene 86, …
- Kautz & McCool 99, McCool et al. 01
- Cabral et al. 99
- Latta and Kolb 02

Frequency Space Methods (spherical harmonics)

- Cabral et al. 87, Sillion et al. 91, Westin et al. 92
- Ramamoorthi & Hanrahan 01
- Basri & Jacobs 01

OLF Factorization

$$B(\vec{R},\vec{N},\vec{V}) : f(\vec{R})g(\vec{N})h(\vec{V})$$

$$B(\vec{R},\vec{N}) : f(\vec{R})g(\vec{N})$$

Advantages

- Naturally captures diffuse, reflective parts

Latta and Kolb 02
Wood et al. 00

$$B(\vec{R},\vec{V}) : f(\vec{R})h(\vec{V})$$

Advantages

- Good param. for *both* BRDF, C
- Fast computation with convolu
- Single reflection map for each